2024 WRAPPED

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University of Luxembourg

15th January 2025

Accomplishment in 2024

Publications:

- **Left adjoint to precomposition in elementary doctrines**, *Theory and Applications of Categories*, Vol. 41, 2024, No. 15, pp 493-515 (from chapter 4 of my PhD thesis, published in May);

- **Adding a constant and an axiom to a doctrine**, *Mathematical Logic Quarterly*, Vol. 70, Issue 3, pp 294–332, 2024 (from chapter 2 of my PhD thesis, published in October).

Preprints:

- **Quantifier-free formulas and quantifier alternation depth in doctrines**, with Marco Abbadini;

- **Freely adding one layer of quantifiers to a Boolean doctrine**, with Marco Abbadini.

Accomplishment in 2024

Talks:

- **Henkin's Theorem for doctrines**, Groupe de travail "Semantique", IRIF - February 7th, Paris;
- **Quantifier alternation depth in universal Boolean doctrines**, with Marco Abbadini, Local seminar of the Logic group, Universita' degli Studi di Padova - 2nd May, Padua;
- **Quantifier-free formulas and quantifier alternation depth in doctrines**, 5th ItaCa Workshop, Universita' degli Studi di Padova - December 19th, Padua.

Accomplishment in 2024

Conferences attended:

- **Abstract Interpretation Workshop**, University of Luxembourg, June 19-21;
- **CT&CatAlg** - *Workshop on the occasion of Enrico Vitale 60th birthday*, University of Milan, September 9-10;
- **ECCL24** - *A workshop dedicated to Pino Rosolini on the occasion of his 70th birthday*, University of Genoa, September 12-13;
- **ItaCa** - *5th ItaCa Workshop*, University of Padua, December 19-20.

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Teaching:

- (Conclusion of) exercise session for **Linear Algebra 1** - Mathematics (head of the course: I. Nourdin);
- Office Hours for **Linear Algebra 1** - Computer Science and Physics (head of the course: A. Perucca).

My research

$$\begin{array}{rcl} \text{Boolean algebras} & : & \text{Classical propositional logic} \\ & = & \\ \text{First-order Boolean doctrines} & : & \text{Classical first-order logic} \end{array}$$

$$\mathsf{Ctx}^{\mathrm{op}} \longrightarrow \mathsf{BA}$$

$$\vec{x} \longmapsto \mathrm{WFF}(\vec{x}) \qquad \ni \alpha(\vec{x})$$

$$(x) \qquad \mathrm{WFF}(x) \qquad \ni (x = 1) \vee (x \geq 2)$$

My research

Boolean algebras	:	Classical propositional logic
	=	
First-order Boolean doctrines	:	Classical first-order logic

$$\mathsf{Ctx}^{\mathrm{op}} \longrightarrow \mathsf{BA}$$

$\vec{x}$	$\mathrm{WFF}(\vec{x})$	$\ni \alpha(\vec{x})$
$\vec{y}$	$\mathrm{WFF}(\vec{y})$	
(x)	$\mathrm{WFF}(x)$	$\ni (x = 1) \vee (x \geq 2)$
(y_1, y_2)	$\mathrm{WFF}(y_1, y_2)$	

My research

Boolean algebras	:	Classical propositional logic
	=	
First-order Boolean doctrines	:	Classical first-order logic

$$\mathsf{Ctx}^{\mathrm{op}} \longrightarrow \mathsf{BA}$$

$$\vec{x} \qquad \mathrm{WFF}(\vec{x}) \qquad \ni \alpha(\vec{x})$$

$$\vec{t}(\vec{y}) \qquad [\vec{t}(\vec{y})/\vec{x}]$$

$$\vec{y} \qquad \mathrm{WFF}(\vec{y}) \qquad \ni \alpha(\vec{t}(\vec{y})/\vec{x})$$

$$(x) \qquad \mathrm{WFF}(x) \qquad \ni (x = 1) \vee (x \geq 2)$$

$$y_1 + y_2$$

$$(y_1, y_2) \qquad \mathrm{WFF}(y_1, y_2) \qquad \ni (y_1 + y_2 = 1) \vee (y_1 + y_2 \geq 2)$$

My research

Quantifiers define Galois connections!

$$\begin{array}{ccc} \mathrm{WFF}(x,y) & \underset{\exists x}{\overset{\text{dummy}}{\leftrightarrows}} & \mathrm{WFF}(y) \\ \beta(y) & \longleftarrow\!\shortmid & \beta(y) \\ \alpha(x,y) & \longmapsto & \exists x \alpha(x,y) \end{array} \qquad \begin{array}{ccc} \mathrm{WFF}(y) & \underset{\text{dummy}}{\overset{\forall x}{\leftrightarrows}} & \mathrm{WFF}(x,y) \\ \forall x \alpha(x,y) & \longleftarrow\!\shortmid & \alpha(x,y) \\ \beta(y) & \longmapsto & \beta(y) \end{array}$$

$$\exists x \alpha(x,y) \vdash \beta(y) \iff \alpha(x,y) \vdash \beta(y) \qquad \beta(y) \vdash \alpha(x,y) \iff \beta(y) \vdash \forall x \alpha(x,y)$$

My research

	Boolean algebras	:	Formulas in a propositional language wrt a theory
		=	
	First-order Boolean doctrines	:	Fmlas in a predicate language wrt a f.-o. theory
		=	
new[1]:	Boolean doctrines	:	Quantifier-free fmlas modulo a f.-o. theory
		=	
to do:	???	:	Sets $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \cdots \subseteq \mathcal{F}_n$ of fmlas with QAD $\leq n$ modulo a f.-o. theory

[1] jww Marco Abbadini, University of Birmingham

My research

	Boolean algebras	:	Formulas in a propositional language wrt a theory
		=	
	First-order Boolean doctrines	:	Fmlas in a predicate language wrt a f.-o. theory
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new[1]:	Boolean doctrines	:	Quantifier-free fmlas modulo a f.-o. theory
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to do:	???	:	Sets $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \cdots \subseteq \mathcal{F}_n$ of fmlas with QAD $\leq n$ modulo a f.-o. theory

Consequence: algebraic proof of ***Herbrand's Theorem*** in f.-o. logic that also allows for the empty model.

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My research

Take further steps in Abstract Constraint Programming, using the formalization given by lattice theory:

- we found some sufficient conditions on the abstract domain in order to have a Bound Abstraction $(X \to \mathscr{P}(\mathbb{U}), \dot{\subseteq}) \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} (X \to L, \dot{\leq})$, with $L \subseteq \mathbb{U}$;
- we interpreted some function symbols in the abstract domain of important examples (e.g. Integers Upper/Lower Bound Abstraction, Floating-point Upper/Lower Bound Abstraction, their correspondent Interval Abstraction...);
- to do: interpret some predicate symbols in the same abstract domains;
- to do: define new abstractions from existing ones (e.g. using Hoare/Smyth construction).

Goals for 2025

- Find a postdoc :)

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- Find a postdoc :)
- with Pierre: finish our paper on Abstract Constraing Programming;
- with Marco Abbadini: correct our first preprint according to the advices of the referee, and continue to characterise the layers of QAD;
- improve and resubmit the preprint from chapter 3 of my PhD thesis.