

# 2024 WRAPPED

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University of Luxembourg



15<sup>th</sup> January 2025

## Accomplishment in 2024

### Publications:

- **Left adjoint to precomposition in elementary doctrines**, *Theory and Applications of Categories*, Vol. 41, 2024, No. 15, pp 493-515 (from chapter 4 of my PhD thesis, published in May);
- **Adding a constant and an axiom to a doctrine**, *Mathematical Logic Quarterly*, Vol. 70, Issue 3, pp 294–332, 2024 (from chapter 2 of my PhD thesis, published in October).

### Preprints:

- **Quantifier-free formulas and quantifier alternation depth in doctrines**, with Marco Abbadini;
- **Freely adding one layer of quantifiers to a Boolean doctrine**, with Marco Abbadini.

## Accomplishment in 2024

### Talks:

- **Henkin's Theorem for doctrines**, Groupe de travail "Semantique", IRIF - February 7th, Paris;
- **Quantifier alternation depth in universal Boolean doctrines**, with Marco Abbadini, Local seminar of the Logic group, Universita' degli Studi di Padova - 2nd May, Padua;
- **Quantifier-free formulas and quantifier alternation depth in doctrines**, 5th ItaCa Workshop, Universita' degli Studi di Padova - December 19th, Padua.

## Accomplishment in 2024

Conferences attended:

- **Abstract Interpretation Workshop**, University of Luxembourg, June 19-21;
- **CT&CatAlg** - *Workshop on the occasion of Enrico Vitale 60th birthday*, University of Milan, September 9-10;
- **ECCL24** - *A workshop dedicated to Pino Rosolini on the occasion of his 70th birthday*, University of Genoa, September 12-13;
- **ItaCa** - *5th ItaCa Workshop*, University of Padua, December 19-20.

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### Teaching:

- (Conclusion of) exercise session for **Linear Algebra 1** - Mathematics (head of the course: I. Nourdin);
- Office Hours for **Linear Algebra 1** - Computer Science and Physics (head of the course: A. Perucca).

Boolean algebras : Classical propositional logic  
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First-order Boolean doctrines : Classical first-order logic

$\text{Ctx}^{\text{op}} \longrightarrow \text{BA}$

$\vec{x} \longmapsto \text{WFF}(\vec{x}) \ni \alpha(\vec{x})$

$(x) \quad \text{WFF}(x) \ni (x = 1) \vee (x \geq 2)$

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$\text{Ctx}^{\text{op}} \longrightarrow \text{BA}$

$\vec{x}$        $\text{WFF}(\vec{x})$        $\ni \alpha(\vec{x})$

$\vec{y}$        $\text{WFF}(\vec{y})$

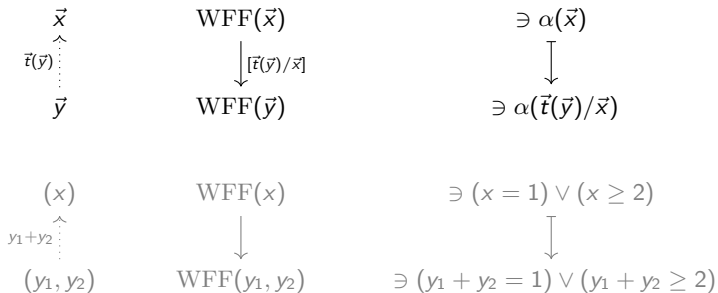
$(x)$        $\text{WFF}(x)$        $\ni (x = 1) \vee (x \geq 2)$

$(y_1, y_2)$        $\text{WFF}(y_1, y_2)$

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$\text{Ctx}^{\text{op}} \longrightarrow \text{BA}$





Quantifiers define Galois connections!

$$\text{WFF}(x, y) \begin{array}{c} \xleftarrow{\text{dummy}} \\ \xrightarrow{\exists x} \end{array} \text{WFF}(y)$$

$$\beta(y) \longleftarrow \beta(y)$$

$$\alpha(x, y) \longmapsto \exists x \alpha(x, y)$$

$$\exists x \alpha(x, y) \vdash \beta(y) \iff \alpha(x, y) \vdash \beta(y)$$

$$\text{WFF}(y) \begin{array}{c} \xleftarrow{\forall x} \\ \xrightarrow{\text{dummy}} \end{array} \text{WFF}(x, y)$$

$$\forall x \alpha(x, y) \longleftarrow \alpha(x, y)$$

$$\beta(y) \longmapsto \beta(y)$$

$$\beta(y) \vdash \alpha(x, y) \iff \beta(y) \vdash \forall x \alpha(x, y)$$

	Boolean algebras	:	Formulas in a propositional language wrt a theory
		=	
	First-order Boolean doctrines	:	Fmlas in a predicate language wrt a f.-o. theory
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new <sup>1</sup> :	Boolean doctrines	:	Quantifier-free fmlas modulo a f.-o. theory
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to do:	???	:	Sets $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \dots \subseteq \mathcal{F}_n$ of fmlas with QAD $\leq n$ modulo a f.-o. theory

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<sup>1</sup>jww Marco Abbadini, University of Birmingham

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Consequence: algebraic proof of *Herbrand's Theorem* in f.-o. logic that also allows for the empty model.

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Take further steps in Abstract Constraint Programming, using the formalization given by lattice theory:

- we found some sufficient conditions on the abstract domain in order to have a Bound Abstraction  $(X \rightarrow \mathcal{P}(\mathbb{U}), \dot{\subseteq}) \xleftrightarrow[\alpha]{\gamma} (X \rightarrow L, \dot{\leq})$ , with  $L \subseteq \mathbb{U}$ ;
- we interpreted some function symbols in the abstract domain of important examples (e.g. Integers Upper/Lower Bound Abstraction, Floating-point Upper/Lower Bound Abstraction, their correspondent Interval Abstraction...);
- to do: interpret some predicate symbols in the same abstract domains;
- to do: define new abstractions from existing ones (e.g. using Hoare/Smyth construction).

## Goals for 2025

- Find a postdoc :)

## Goals for 2025

- Find a postdoc :)
- with Pierre: finish our paper on Abstract Constraining Programming;
- with Marco Abbadini: correct our first preprint according to the advices of the referee, and continue to characterise the layers of QAD;
- improve and resubmit the preprint from chapter 3 of my PhD thesis.