Quantum Circuit Optimization using ZX Calculus.

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Quantum computing

Applications

- Factorization
  - Shor [21] $\mathcal{O}(\log N^3)$ vs. GNFS [17] $\mathcal{O}(\exp\sqrt{\frac{64}{9}}\log N^{\frac{1}{3}}\log\log N^{\frac{2}{3}})$
- Unstructured search
  - Grover [8] $\mathcal{O}(\sqrt{N})$ vs. linear search [12] $\mathcal{O}(N)$

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  - Molecular interaction [1]
- Quantum artificial intelligence
  - Perovskite structure prediction [16]
  - Climate modelling [25]

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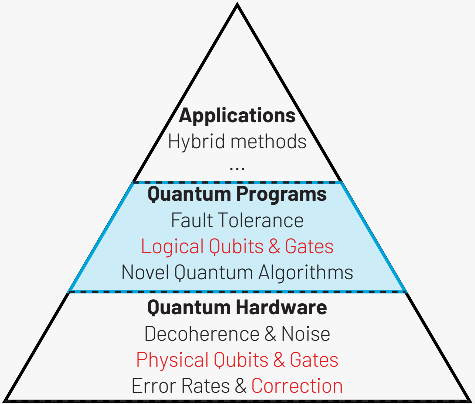
Cool. What is the catch?

Quantum computing

Current challenges in quantum computing

- **Ressource restrictions**:
  - 127 logical qubits [6]
  - up to ≈ 5000 logical gates
  - short coherence time ($80[\mu s]$ to $1[ms]$) [22]
- **Error correction**:
  - noise drives gate error rate [24]
  - limits the number of usable gates
  - overhead vary by order of magnitude for different gates [20]
- **Quantum computing limited to artificial problems**

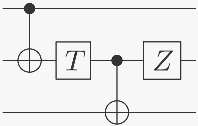


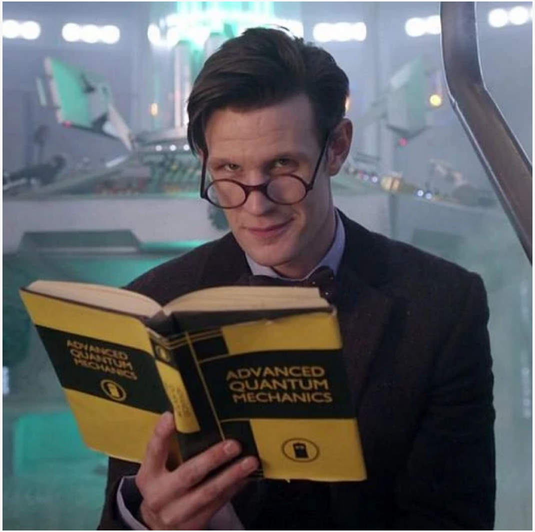
$\Rightarrow$ **Architecture-independent QC optimization**

Quantum Computing

Quantum Circuits [18]

- Analagous to classical logic gates
- But **reversible**
- Input is reconstructable from output
- Not all gates have classical counter part (eg. Hadamard)





QC optimization

- Infinite universal gate sets
- Infinite gate commutation rules
- Equivalence verification computational expensive

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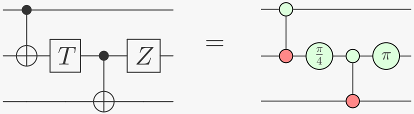
$\Longrightarrow$

ZX calculus [2, 3]

- 8 generators
- 9 well defined rewriting rules
- Rewriting rules preserve semantics



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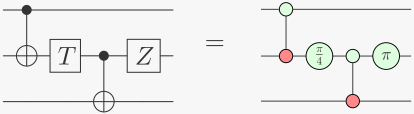
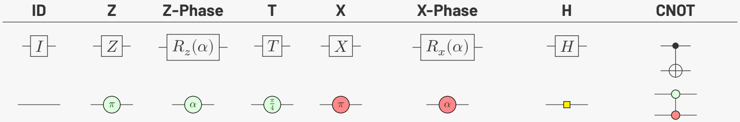


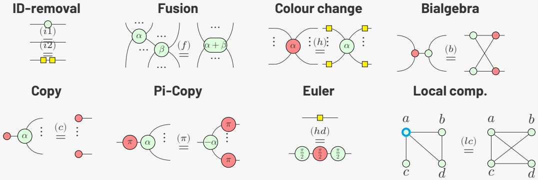
$\Rightarrow$ **State-space for combinatorial optimization is (infinitely) large**

ZX calculus

Diagrammatic Reasoning Framework



- Every QC can be expressed as a ZX diagram [26]
- **Semantic preserving rewriting rules**
- Circuit extraction is # P-hard [4]
- Applied for QC optimization and verification



ZX calculus

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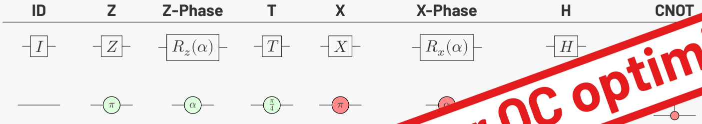

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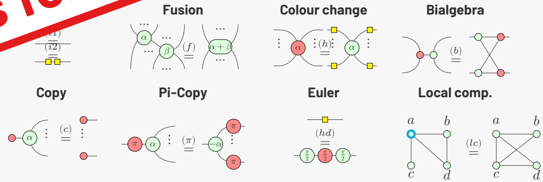

How can use ZX Calculus for QC optimization?

Contributions - Foundation of Exhaustive Search

2024 - Catch up to SOTA

- **Formalization** of ZX diagram optimization
- New set of **pruning conditions** for state-space reduction
- Reproducible **framework** that integrates in standard quantum compilation pipelines (≈ 7000 LOC)
- **Exhaustive search**:
  - **Equals T-gate count of SOTA** on 89% of the instances
  - **Reduce the edge count** by 22% on average **close to SOTA** (29%)
- Novel local elimination **state-space search** (TBC)

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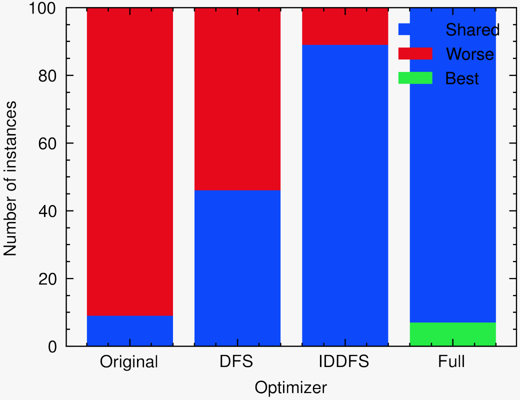


Figure: T-gate count after 1.5 hours

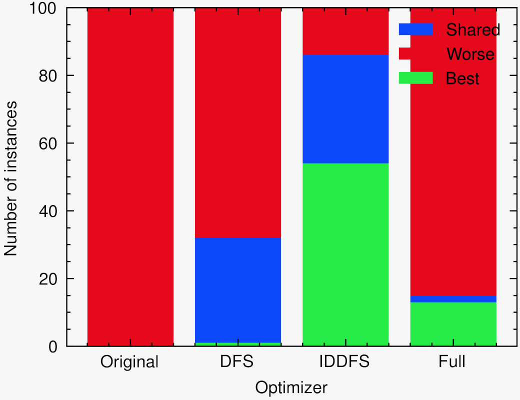
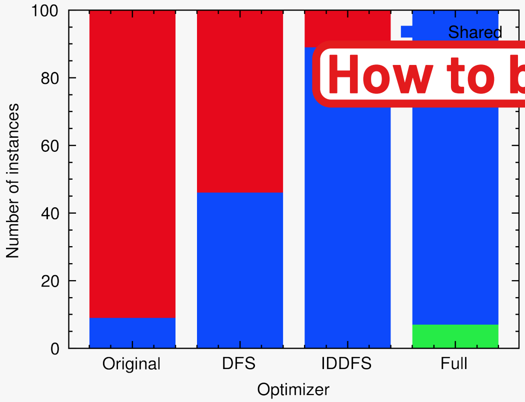


Figure: Edge count after 1.5 hours



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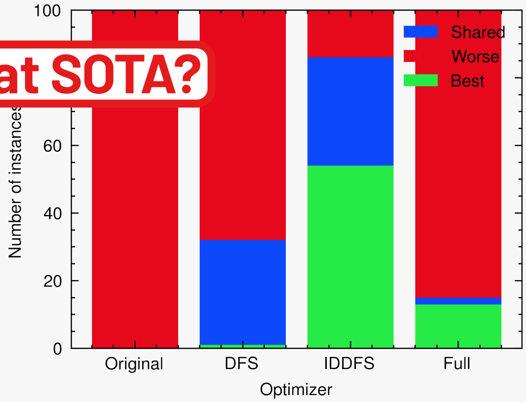

Figure: Edge count after 1.5 hours

How to beat SOTA?

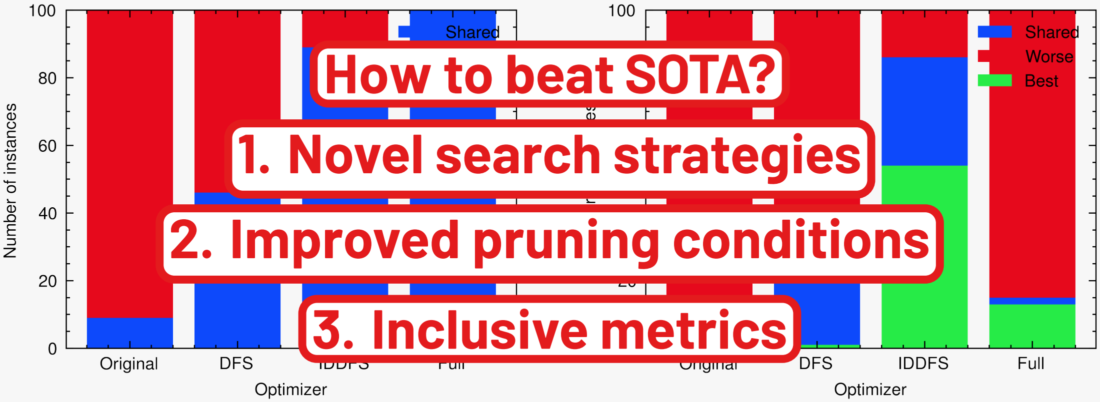

Figure: T-gate count after 1.5 hours

Figure: Edge count after 1.5 hours

Contributions - Improved Search for Quantum Circuit Optimization using ZX Calculus

2025 - Beat SOTA?

- !! Improved search algorithms (Journal Paper)
  - extends OLA 2025
  - adds novel local elimination search
  - includes limited discrepancy search
- ! Survey
- Improved Metrics
  - Edge count $\leftrightarrow$ Two-qubit gate count
  - Lexiographic optimization (eg. T-gate count $\rightarrow$ Edge count $\rightarrow$ Two-qubit gate count)
- Pruning conditions (allow unfusion; single rule application)

Advancing Quantum Circuit Optimization through ZX Calculus and Exhaustive Search I

1. Introduction
2. Preliminaries
   - 2.1 Introduction to ZX Calculus
   - 2.2 ZX Optimization
3. ZX Diagram Optimization
   - 3.1 Definition of a ZX Diagram
   - 3.2 Definition of a Quantum Circuit
   - 3.3 Formalization of ZX Diagram optimization
   - 3.4 State space formed by different search algorithms
     - 3.4.1 DFS & IDDFS
     - 3.4.2 LDS
     - 3.4.3 Local elimination
   - 3.5 Pruning conditons
   - 3.6 Metrics

Advancing Quantum Circuit Optimization through ZX Calculus and Exhaustive Search II

Conclusion

- Caught up to SOTA in 2024
    - Formalization
    - Framework implemented
    - Equate T-gate count on 89% of the instances within 1.5 hours
- Let's beat SOTA in 2025!
    - Local discrepancy search
    - Novel local elimination search

Conclusion

- Caught up to SOTA in 2024
  - Formalization
  - Framework implemented
  - Equate T-gate count on 89% of the instances within 1.5 hours
- Let's beat SOTA in 2025!

You will rock your research goals in 2025!

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